Development of a Three-Dimensional Upwind Parabolized Navier-Stokes Code

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Abstract

THE design of future hypersonic flight vehicles will depend heavily on computational fluid dynamics for the prediction of aerodynamic and thermodynamic loads as well as engine performance. One of the features that characterizes the hypersonic flow regime is the presence of strong shock waves generated by the vehicle and by protuberances from the main body such as wings, canopies, and engine inlets. Thus, a need exists for a robust computational tool that can efficiently and accurately resolve flowfields containing discontinuities.

In the present work, an upwind algorithm designed for the integration of the parabolized Navier-Stokes (PNS) equations[1] has been extended to three dimensions. Conventional PNS solvers[2,3] are based on the central differencing of crossflow fluxes and, therefore, have difficulty capturing strong embedded shocks. The objective of this work is to mate the efficiency of the space-marching approach with the advantageous shock-capturing characteristics of upwind schemes. In addition to being upwind, the new algorithm is implicit (including boundary conditions) and is based on the use of finite volumes for accurate flux conservation. The new code is applied to laminar hypersonic flows past two simple body shapes: a circular cone of 10-deg half-angle and a generic all-body hypersonic vehicle. Cone flow solutions were computed at angles of attack of 12, 20, and 24 deg, and results are in good agreement with experimental data. Results also were obtained for the flow past the all-body vehicle at angles of incidence of 0 and 10 deg.

Contents

The PNS equations, obtained from the steady Navier-Stokes equations by dropping the streamwise viscous derivatives separating the streamwise pressure gradient, can be written with respect to a generalized coordinate system as

$$\frac{\partial \bar{E}^*}{\partial \xi} + \frac{\partial \bar{E}^p}{\partial \xi} + \frac{\partial(\bar{F}_i - \bar{F}_v)}{\partial \eta} + \frac{\partial(\bar{G}_i - \bar{F}_v)}{\partial \zeta} = 0 \tag{1}$$

where $\bar{F}$ and $\bar{G}$ present fluxes in the η and ζ coordinate directions, respectively. The subscripts i and v indicate inviscid and viscous components, respectively. The first two terms of Eq. (1) arise from the use of the Vigneron technique[2] in which the streamwise flux vector is split into two parts: the first part ($\bar{E}^*$) is the modified streamwise flux, and the remainder ($\bar{E}^p$) is that part of the streamwise pressure gradient responsible for introducing ellipticity into the equations in subsonic regions. The latter part is usually neglected or treated as a source term so that Eq. (1) becomes hyperbolic-parabolic in nature.

Equation (1) is differenced in a finite-volume manner to yield the following discretized conservation law:

$$\begin{aligned}\hat{A}^{*n}_{k,l}\delta^{n+1}U_{k,l} = &-(\hat{A}^{*n}_{k,l} - \hat{A}^{*n-1}_{k,l})U^n_{k,l} \\ &-[(\hat{F}_i - \hat{F}_v)^{n+½}_{k+½,l} - (\hat{F}_i - \hat{F}_v)^{n+½}_{k-½,l}] \\ &-[(\hat{G}_i - \hat{G}_v)^{n+½}_{k,l+½} - (\hat{G}_i - \hat{G}_v)^{n+½}_{k,l-½}] \\ &-[\hat{E}^p(dS^{n+1}_{k,l}, U^n_{k,l}) - \hat{E}^p(dS^n_{k,l}, U^{n-1}_{k,l})]\end{aligned} \tag{2}$$

where

$$U = [\rho, \rho u, \rho v, \rho w, E_t]^T$$

$$\hat{A}^{*n-1} = \frac{\partial \hat{E}^*(dS^n, U^{n-1})}{\partial U^{n-1}}$$

The left-hand side and the first term on the right-hand side of Eq. (2) result from the linearization of the streamwise flux vector

$$\hat{E}^*(dS^n, U^n) = \hat{A}^{*n-1}U^n$$

which is applied in order to avoid the difficulty of extracting flow properties from $\hat{E}^*$. This linearization also simplifies the application of the implicit algorithm. The indices on the arguments dS and U indicate the location where the geometry and the physical variable, respectively, are evaluated.

A conventional central-differencing scheme would be obtained from Eq. (2) by simply averaging flow properties from adjacent cell faces. Upwind schemes derive their superior shock-capturing characteristics from the introduction of flow physics at this level of the algorithm. In the present algorithm, the inviscid fluxes in the crossflow directions are evaluated through the solution of the approximate form of the governing equations

$$\frac{\partial \hat{E}^*}{\partial \xi} + D_{m+½}\frac{\partial \hat{E}^*}{\partial \kappa} = 0$$

with initial conditions

$$\hat{E}^{*n}(\kappa) = \begin{cases}\hat{E}^*(dS^n_{m+½}, U_m) & \text{where} \quad \kappa < \kappa_{m+½} \\ \hat{E}^*(dS^n_{m+½}, U_{m+1}) & \text{where} \quad \kappa > \kappa_{m+½}\end{cases}$$

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where

$$D_{m+\frac{1}{2}} = \frac{\partial \hat{F}_i}{\partial \hat{E}^*} \quad \text{or} \quad \frac{\partial \hat{G}_i}{\partial \hat{E}^*}$$

and κ and m are replaced with η and k or ζ and l for the calculations of $\hat{F}_i$ or $\hat{G}_i$, respectively. This problem is a steady form of Roe's approximate Riemann problem,[4] and the flow properties making up $D_{m+\frac{1}{2}}$ are averaged between m and $m+1$ using standard Roe averaging.

The solution to the above approximate Riemann problem consists of four constant-property regions separated by three surfaces of discontinuity emanating from the cell edge (ξ^n, $\kappa_{m+\frac{1}{2}}$) and having slopes given by the eigenvalues of $D_{m+\frac{1}{2}}$. Of particular interest to the numerical algorithm is the resulting flux across the $m + \frac{1}{2}$ cell interface. This first-order-accurate inviscid flux consists of an unbiased component and a first-order upwind dissipation term as follows:

$$H^I_{m+\frac{1}{2}} = H^c - \tfrac{1}{2}|D|_{m+\frac{1}{2}}[\hat{E}^*(dS^n_{m+\frac{1}{2}}, U_{m+1}) - \hat{E}^*(dS^n_{m+\frac{1}{2}}, U_m)] \tag{3}$$

where H^c is a simply averaged flux, and $|D|$ is the matrix that has the same eigenvectors as D and has eigenvalues that are the absolute values of those of D. First-order inviscid numerical fluxes in the η and ζ directions then are given by

$$(\hat{F}^I_i)_{k+\frac{1}{2},l} = H^I_{k+\frac{1}{2},l}$$

$$(\hat{G}^I_i)_{k,l+\frac{1}{2}} = H^I_{k,l+\frac{1}{2}}$$

respectively, where κ in Eq. (3) is replaced by η or ζ accordingly.

The scheme is extended to second-order accuracy following the approach of Chakravarthy and Szema[5] in which a second-order generic numerical flux is defined in terms of the first-order flux and antidissipative correction terms. The added terms are limited relative to one another in order to eliminate overshoots and undershoots that are characteristic of second-order schemes. In order to eliminate nonphysical behavior at locations where eigenvalues change sign, local dissipation is added in regions where eigenvalues are small in magnitude.

The algorithm is made implicit by evaluating the first-order numerical flux at the $n+1$ marching station and lagging the second-order correction terms at the nth level. A straightforward linearization in ξ is then applied assuming the $|D|$ matrix is locally frozen. The resulting system of algebraic equations is factored in a conventional manner (e.g., see Refs. 2 and 3) to produce an alternating direction implicit scheme of the form

$$\left[\hat{A}^*_{k,l} + \frac{\partial(\delta_\eta\{\hat{F}^I_i - \hat{F}_v\})}{\partial U_{k,l}} + \bar{\delta}_\eta\left(\frac{\partial\{\hat{F}^I_i - \hat{F}_v\}}{\partial U}\right)\bullet\right]^n \times [(\hat{A}^*_{k,l})^{-1}]^n \left[\hat{A}^*_{k,l} + \frac{\partial(\delta_\zeta\{\hat{G}^I_i - \hat{G}_v\})}{\partial U_{k,l}} + \bar{\delta}_\zeta\left(\frac{\partial\{\hat{G}^I_i - \hat{G}_v\}}{\partial U}\right)\bullet\right]^n \delta^{n+1}U_{k,l} = RHS^n \tag{4}$$

where RHS^n is of the same form as the right-hand side of Eq. (2) except that the crossflow fluxes are evaluated using flow properties at the nth marching level.

Because of the Vigneron technique, the eigenvalues of D may obtain values as large as 10^9 in the subsonic region when large step sizes are being taken. The associated eigenvector matrices are highly ill-conditioned, and computer roundoff errors can be amplified to the point where they destabilize the calculations. In the present version of the code, this is avoided by simply deactivating the upwinding in the subsonic region. Efforts are in progress that would allow a modified upwind dissipation term to be included near the wall.

Results

In order to evaluate the performance of the new algorithm in solving the three-dimensional PNS equations, flows past two simple geometries were computed. The first flow-field computed with the new code was that of Mach 7.95 laminar flow past a 10 deg half-angle circular cone. The flow conditions were chosen to match those investigated experimentally by Tracy.[6] Results illustrated in the full paper show predictions of flow-field geometries and surface pressures that are in good agreement with experimental data. The second test case consisted of Mach 7.4 flow past an all-body hypersonic aircraft. These calculations are being performed in conjunction with an experimental investigation of this geometry in the 3.5-ft hypersonic wind tunnel at NASA Ames Research Center. Preliminary comparisons indicate good agreement with experimental surface pressures and pitot pressure profiles. Comparisons with experimental heat transfer data for these geometries are currently under study; heat transfer predictions for other configurations including a blunt bicone[7] and a generic hypersonic vehicle have been obtained and show good agreement with experiment.

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